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Estimating the optimal number of servers for an emergency department of a private hospital, via mathematical programming.

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Abstract: There is a great need for administrators of private organizations, as an organization seeking for profit, to be familiar with the efficient methods of allocating scarce resources. On the other side they are seeking for quality which can be measured in terms of several criteria. In health care organizations quality can be measured in terms of the number of admissions, discharges, ease of access to medical care, the number of procedures needed from the patient in the health care service and the organization profit. While quality can be measured, from the patient point of view, in terms of, duration of stay in the hospital, because no patient likes to wait, especially when he is served in an emergency department and also he aim to minimize the cost of service. At the emergency department of a hospital as a system of immediate concern, it is important to minimize the critical time or the so called response time for presenting the patient service. The main objective of this paper is to apply the mathematical programming and in particular goal programming (GP) with the aid of queuing theory and the data envelopment analysis (DEA) to determine the optimal number of physicians and nurses in order to serve emergency department patients. The model solution helps the decision maker to, efficiently; allocate the scarce resources of a private hospital and to realize as high level of organization quality as possible.

Keywords: allocation problem.Emergency department.Goal programming. Queuing theory.Data envelopment analysis

INTRODUCTION:

Health care resources are always limited as compared with the demands for efficient services that are frequently needed by the customers (patients). On the opposite side hospital facilities , which are unoccupied or unutilized, are a buffer against the risk of not having the needed facilities when the demand for them increases, i.e. high quality is often associated with high payment (cost), although costs must be kept within certain limits. The costs of that buffer are the cost of:

1) Construction, set up
2) Maintaining unoccupied facilities.
3) Staff salaries who are partly idle when not needed.

Emergency department (ED) can be considered as the preface of any private hospital. Failure in accepting or providing health care for some patients may cause a big damage in the hospital reputation. The response time is very important issue for an ED patient, which is defined as the time between a patient arrival at the hospital and the time he meet a physician to provide the suitable care needed and it can be measured by the service rate. Measuring efficiency is a main issue for any serious organization.

Cases of an emergency patient can be divided into:

1. No severe case: where there is serious injury for the patient so he can be fixed and turned away.
2. Severe case: where a patient must turn in to a serious and intensive care to provide a long treatment.

ED is staffed with, say C, physicians of different specialty and, say n, nurses, who have two tasks, registering tasks and helping physicians during the shift. Also ED has say R beds available to serve patient who may be investigated and turned out or turned in as an inpatient when his case is severe.

Determining the optimal number of physicians and nurses helps the DM`s to ,efficiently, allocate the scarce resources and to meet the requirements of the patient flow ,i.e, to realize some or all of the following conflicting objectives:

- ✓ Minimizing allocation costs.
- ✓ Maximizing the income resulting from medical service.
- ✓ Decreasing waiting times for the patients which are a measure of quality.
- ✓ Maximizing the system efficiency by better usage of the human and spatial availability.
- ✓ Maximizing the busy number (utilization) of working staff, per shift.

Many of these objectives may have different importance while conflicting ones. They may compete for scarce resources.

Mc Quarrie (1983) has presented a study to find a range for maximal hospital occupancy rates using various queuing systems. Panayiotopoulos (1984) studied a hospital emergency department using a general simulation algorithm via queuing systems .Cooper (1974) presented a mathematical method to estimate the optimal number of beds in the cardiac-care services of a hospital. Claire et al, (2010) have developed a two stage stochastic mathematical programming formulation to allocate resources between health care programs when there is an exogenous budget and the parameters of the health care models are variable and uncertain. Epstein et al,(2008) provided a general mathematical programming model for health care allocation that allows to incorporate important aspects that are not available if the decision making is made using threshold values of incremental cost-effectiveness ratio alone. Behner and Fogg (1990) have used economic and statistical analysis to formulate and interpret the relationship between service level and nurse stuffing decisions. Sendi, et al (2003) have examined the implications of indivisibility on the mathematical programming results. Stinnett and Paltiel (1996) provided a general mathematical programming framework to accommodate information regarding returns to scale indivisibilities in the patient population, program interdependence and ethical constraints. Beraldi,et al (2004) have contributed a solution for the problem of designing and planning the ED system via stochastic programming approach. Joshi et al (2011), has shown how different arrival patterns and time duration of service can affect the ED`s ability to treat patients during a conventional terror disaster event. They divided patients into 3 types:

1. Patients suffering from injuries that need immediate help i.e. that were found to be closest to the blast point i.e. they have serious injuries.
2. Patients requiring a greater degree of medical care and hospitalization, but not expected to progress to a life threatening status.
3. Patients with minor injuries who are capable of self transporting themselves to the hospital; these patients require basic medical and do not require hospitalization.

Fakrell (2009) has introduced a comprehensive study about phase-type distributions, including the hyper exponential distribution, and gave a survey of where they have been used in the healthcare industry, and proposed some ideas on how they could be further utilized. McClean and Millard [1993], while not specifically referring to *PH* distributions, have fitted the order two hyper exponential distributions to the length of stay of patients in a geriatric medicine department. That is, the density function for the length of stay distribution(t), follows phase-type distributions for $t \geq 0$, μ_{1t}, $\mu_{2t} > 0$, and $0 < \rho < 1$, was given by:

$$f(t) = \rho e{-}\mu_{1t} + (1 - \rho) e{-}\mu_{2t} \quad t \geq 0,$$

where

μ_{1t}: represents the service rate of type 1 patients.

μ_2: represents the service rate of type 2 patients.

ρ: represents the proportion of (either male or female) short stay patients,

Type 1 patients: are Patients with minor injuries i.e., who are capable of self transporting themselves to the hospital; these patients require basic medical and do not require hospitalization.

Type 2 patients: are Patients with major injuries i.e., who required a greater degree of medical care and hospitalization, but not expected to progress to a life threatening status.

They fitted data for male and female separately. The two states in the model represented acute/rehabilitative (short stay) patients (type 1 patients), and long stay patients (type 2 patients). Patients who left the system by either being discharged or dying were categorized as short stay, and those who left by being transferred elsewhere, as long stay. The parameter ρ was estimated by the proportion of (either male or female) short stay patients, and μ_1 and μ_2 were estimated by the reciprocal of the mean length of stay for short and long stay patients, respectively. The model was improved by fitting a mixture of a lognormal distribution (for short stay) and an exponential distribution (for long stay).Laskowski et al (2009) has applied both agent models and queuing theory to investigate patient access and patient flow through emergency departments.

Efficiency can be defined as the decision making unit (DMU) ability to produce the maximum amount of output with a given amount of inputs; or, using minimum amount of inputs to produce a given amount of output. DEA is a relatively data-oriented approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. **DEA** is considered as a nonparametric method in operations research and economics for the estimation of production frontiers. It is used to empirically measure productive efficiency of DMUs.

DMUs are the economic entities or units whose efficiencies could be measured by the DEA; those units should be homogeneous, work in the same field and have the same inputs and outputs variables. Efficiency can be divided into:

- ✓ **Cost Efficiency (CE):** An entity will be cost efficient only if, it is both technically and allocatevely efficient.
- ✓ **Economic Efficiency:** It means, producing the maximum value of outputs with a given value of inputs; or equivalently, using minimum value of inputs to produce a given value of output.
- ✓ **Pareto Efficiency:** A central concept in economics is Pareto efficiency. A situation is said to be Pareto efficient if there is no way to rearrange things to make at least one person better off without making anyone worse off.
- ✓ **Pure Technical Efficiency:** refers to the firm's ability to avoid waste by producing as much output as input usage allows.
- ✓ **Relative Efficiency(RE):** A firm is to be rated as fully (100%) efficient on the basis of available evidence if and only if the performances of other peers do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.
- ✓ **Scale Efficiency (SE):** It measures the DMU's ability to work at its optimal level of operation. This efficiency affects and contributes to the DMU aggregate technical efficiency.

Charnes, Cooper (1993) described DEA as a mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relations, such as the production functions and/or efficient production possibility surfaces that are cornerstones of modern economics. Asmilda et al [2007] have presented an overall measurement for efficiency and effectiveness using DEA. Rowena et al[2007] have suggested a new measure efficiency in Health Care using the DEA.

Multi objective programming (MOP) is one way of considering multiple objectives explicitly in a mathematical programming framework. It took a widely interest in operation researches since 1970. Reasons for such increasing interest are:

- The increasing recognition that most decision problems are inherently multi- objective.
- Even many problems addressed by normal single-objective models can easily be viewed as multi objective ones, such as project management, inventory and scheduling problems.

Hwang and Masud (1979) have classified solution techniques for MOP problems according to the timing of the requirement for preference information versus the optimization. Goal programming belongs to one of three techniques that depend on prior articulation of the decision makers` preferences.

Goal programming is developed by Charnes and Cooper (1961,1977) to provide the DM`s with the opportunity to satisfy or optimize several diversified goals simultaneously. Lee (1973) has applied the model for problems in healthcare field showing the flexibility of choosing priorities of the goals as a great advantage since it permits to easily choose between different choices to find the best one.

Queuing theory is used for many stochastic environment applications where queues for service reflect the restrictions on the amount of service that can be provided at any one time. Delays in service usually occur when the flow of customers requiring service in a given time interval exceeds the service ability that can be provided in that interval. Congestion, normally, happens in any regular hospital but especially in an ED. The paper aims formulating the allocation of servers of a ED for minimizing the cost of hiring the optimal number of servers in an ED , maximizing efficiency of the ED and maximizing the utilization of the available servers, using GP technique with the aid of queuing theory and the DEA.

2) ED as a queuing system:

In case of ED system with two types of patients, it seems logical to consider two negative exponential phases for service times in parallel. The ED system is fed by a single queue with random arrivals at rate λ and hyper-exponential distribution of the service time with total mean time μ .We have two exponential distributions of the service with rates μ_1 (for no severe patient case) and μ_2 (for severe patient case).When an ED patient enters service, he is assigned to one of the two service 1 or 2 depending on his case. Therefore ED can be considered as an M/H/C:$\infty/FIFO$

where:

M: means that the arrival pattern is "Poisson" arrival distribution (i.e., the distribution of the time between two arrivals is exponential)

H: hyper-exponential distribution of the service time where we have two service rates μ_1 (for no severe patient case) and μ_2 (for severe patient case).

C: the current number of physicians already allocated in the system.

∞ : means infinite capacity of patients.

FIFO: means, the system of service is first in first served,.

Let:

f(t) = be the probability that a unit will complete its service in a time between t and t+Δt:

f(t) = $\sigma\mu_1 e^{-\mu_1 t} + (1-\sigma)\mu_2 e^{-\mu_2 t}$

σ: *is the proportion of patients of no severe case.*

Then the overall mean service rate, μ will be:

$$\mu = \frac{\mu_1\mu_2}{\sigma\mu_2 + (1-\sigma)\,\mu_1}$$

3) Goal programming model of an ED department of a private hospital:

Let

C_{ijk}: be the number of physicians with specialization i ,i =1,2,..,I at shift j, j=1,2,3 while the patient case is k, k=1,2.

i: physician specialty , i=1,2,…,I

j: shift number , j=1,2,3

k: patient`s case(severe or not severe) , k=1,2

a_{ijk}: cost of hiring one physician of type i at shift j for patient case k in an ED ,i=1,...,I j=1,2,3 ,k=1,2.

n_j : needed number of nurses at shift j.

(Notice that there are no specializations for nurses)

g_j : cost of hiring a nurse at shift j.

R: the available number of beds for the ED zone.

The objectives of the model can be derived as follows:

The <u>first objective</u> of the model aims at minimizing the total cost of hiring both physicians and nurses in the ED ,i.e.:

$$\min.[\sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk}a_{ijk} + \sum_{j=1}^{3} g_j n_j]$$

The <u>second objective</u> of the model is to maximize the percentage of busy physicians in the ED per shift:

let

C_{ijk}^{b} : be the number of busy physicians with specialization i at shift j and for patient case k.

The facility utilization of the ED is defined by the following percentage:

The facility utilization = $\sum_{i=1}^{I}\sum_{j=1}^{3}\frac{C_{ijk}^{b}}{C_{ijk}}$ k=1,2

i.e, we have 2 facility utilizations according to the patient`s situation.

Since:

$$\frac{C_{ijk}^{b}}{C_{ijk}} = L_s(ijk) - L_q(ijk) = \frac{\lambda_{ij}}{\mu_{ijk}\, C_{ijk}} \quad \textbf{(2.1)}$$

where:

$L_s(ijk)$: the expected number of patients in the ED system of type k served by a physician with specialization i at shift j.

$L_q(ijk)$: the expected number of patients in the ED queue of type k served by physician with specialization i at shift j.

μ_{ijk}: is the service rate of an ED physician with specialization i at shift j a**nd patient type k .**

λ_{ij}: is the overall arrival rate of the ED system i.e. the arrival rate of the ED system regardless of patient type.

Therefore, summing eq.(2.1) for all values of i,j we can formulate the second objective as follows:

$$max\sum_{i=1}^{I}\sum_{j=1}^{3}\frac{C_{ijk}^{b}}{c_{ijk}} = max\sum_{i=1}^{I}\sum_{j=1}^{3}\frac{\lambda_{ij}}{\mu_{ijk}C_{ijk}} \quad k=1,2$$

Let
$\frac{1}{C_{ijk}} = C`_{ijk}$
Then the second objective will be:

$$\text{Max} \sum_{i=1}^{I}\sum_{j=1}^{3} C`_{ijk}\frac{\lambda_{ij}}{\mu_{ijk}} \quad k=1,2$$

The third objective of the model is to maximize the percentage of busy nurses in the ED per shift:
Following the same procedure used in the second objective for nurses utilization **,can be achieved by:**
Max.$\sum_{i=1}^{I}\sum_{j=1}^{J} n`_j \frac{\lambda`_j}{\mu}$
Where
$n`_j = \frac{1}{n_j}$ j=1,…,3
$\lambda`_j = \frac{\sum_{i=1}^{I}\lambda_{ij}}{I}$ ,i.e., $\lambda`_j$ is the average of the arrival ratesλ_{ij}
.

The Fourth objective of the model is to maximize the efficiency of the ED as follows:

It is well known that efficiency is defined as:

Efficiency of the ED = $\frac{output\ of\ the\ ED}{input\ of\ the\ ED}$, where:

Outputs :can be measured for a hospital department using many types of scales.The ones that will be used here are:
- Number of served patients
- Bed productivity measured by the occupation index.
- Average turnover.

Therefore:
Outputs=$\sum_{p=1}^{3} u_p\ y_p$
Where
y_p: $value\ of\ measure\ p$.i.e,
y_1:is the number of annual served patients in the ED.
y_2: is the annual bed productivity measured as follows:

$$y_2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\frac{\lambda_{ij}}{R} * 360$$

It is calculated as the number of patients arrived to the ED during a year divided by the number of beds available in the ED.
y_3: which is called the occupation index and is calculated as the number of annual admissions divided by the number of beds available in the ED.

u_p : $the\ weight\ of\ measure\ p$. For simplicity we can take u_p as equal weights, i,e., $u_p = 0.33$,p=1,2,3.
Therefore the value of output is well known value to the decision maker so they are considered as constant.

inputs :can be measured using many types of scales.The ones that will be used are:
- Total salaries of physicians
- Total salaries of nurses.

Therefore:

$$\text{Input}=\sum_{q=1}^{2} v_q\, x_q = v_1 \sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk}a_{ijk} + v2\sum_{j=1}^{3} g_j n_j$$

Where
v_q : $are\ unknown\ weights$

Therefore the fourth goal refers to a nonlinear goal and is calculated as follows:

$$\text{Max}\ \frac{output\ of\ the\ ED}{input\ of\ the\ ED} = \frac{\sum_{p=1}^{3} u_p\ y_p}{v_1 \sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk}a_{ijk} + v_2\sum_{j=1}^{3} g_j n_j}$$

Or equivalently:

$$\text{Min}\quad v_1 \sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk}a_{ijk} + v2\sum_{j=1}^{3} g_j n_j$$

,since the nominator is considered as constant.

CONSTRAINTS OF THE MODEL:

1. $\sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk} \le R$
2. $\sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk} \le \sum_{j=1}^{3} nj$
3. $\sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk} \ge C$

Where:
C: is the present number of physicians who currently serve an ED zone.

4. $\sum_{j=1}^{3} nj \ge N$

Where:
N: **is the present number of nurses who currently serve an ED zone.**
The final version of the model:
Lexico.min. A ={ d_1^- , $(d_2^+ + d_3^+ + d_4^+), d_5^-\}$
s.t.

$$\sum_{i=1}^{I}\sum_{j=1}^{3}\sum_{k=1}^{2} C_{ijk} + \sum_{j=1}^{3} gj\, nj + d_1^- - d_1^+ = S$$

$$\sum_{i=1}^{I}\sum_{j=1}^{3} \frac{\lambda_{ij} C_{ij1}}{\mu_{ij1}} + d_2^- - d_2^+ = \mathbf{100}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{3} \frac{\lambda_{ij} C_{ij2}}{\mu_{ij2}} + d_3^- - d_3^+ = \mathbf{100}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{3} n`_j \frac{\lambda`_j}{\mu} + d_4^- - d_4^+ = \mathbf{100}$$

$$v_1 \sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk}a_{ijk} + v2\sum_{j=1}^{3} g_j n_j + d_5^- - d_5^+ = Y$$

$$\sum_{i=1}^{I}\sum_{j=1}^{3}\sum_{k=1}^{2} C_{ijk} \geq C$$

$$\sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{i=1}^{I} C_{ijk} \leq R$$

$$\sum_{i=1}^{I}\sum_{j=1}^{3}\sum_{k=1}^{2} C_{ijk} \leq \sum_{j=1}^{3} n_j \quad \leq \quad \leq$$

$$\sum_{j=1}^{3} nj \geq N$$

Where Y is the outputs constant value ,S is the maximum budget specialized for physicians and nurses salaries.

$C_{ijk} \geq 0, n_j \geq 0$ i=1,…,I , j=1,…,3 ,k=1,…,2

$d_i^-, d_i^+ \geq 0$

$d_i^- * d_i^+ = 0$

The previous model could be solved using a suitable package such as Gams.

CONCLUSION:

Formulating the allocation problem of determining the optimal number of physicians and nurses, who can serve ED patients of a private hospital, assuming two types of patients, is applied using the goal programming method, with the aid of queuing theory and the DEA. The patients' types affect the service time that is given to the patients by the physicians and nurses. The decision maker can, efficiently, allocate the manpower resources, as scarce resources in the ED case, to realize as high level of quality as possible. The model has included the ED efficiency as one of its goals, which reflects the degree of inputs exploitation to produce the desired outputs. Therefore solving the previous model can both optimize the scarce resources and determine the weaknesses of the ED performance.

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